

# Judging 2D versus 3D Square-Wave Virtual Gratings

Peter P. Ho  
UC Berkeley  
Mechanical Engineering Dept.  
2168 Etcheverry Hall  
Berkeley, CA 94720  
peterho@me.berkeley.edu

Bernard D. Adelstein  
NASA Ames Research Center  
Mail Stop 262-2  
Moffett Field, CA 94035  
Bernard.D.Adelstein@nasa.gov

Hami Kazerooni  
UC Berkeley  
Mechanical Engineering Dept.  
5124 Etcheverry Hall  
Berkeley, CA 94720  
kazeroon@me.berkeley.edu

## Abstract

*Haptic virtual texture can enhance people's experience in interacting with virtual objects by providing surface information. We present a study to analyze how people distinguish between 2D and 3D square-wave gratings using a point-source haptic interface. Our analyses were based on objective vibration and force measurements and psychophysical experiments with human subjects. The results indicated that people were unable to detect the difference between the two textures when they moved their hands across textures with either the amplitude or period smaller than 1.52 mm (0.06 inch) for a texture stiffness of 2 N/mm. This result implies that a simple 2-degree-of-freedom haptic interface may be sufficient to convey the same 3-dimensional tactile feeling for certain textures if the textures are small enough.*

## 1. Introduction

The objective of this paper is to help understand how humans distinguish between 2D and 3D textures in haptic virtual environments. In particular, we would like to see whether people can easily discriminate between 2D and 3D square-wave gratings using a point-source haptic interface. Vibration and force exerted on the human hand were measured when the hand was moved across the texture gratings. Furthermore, we asked subjects to do a series of psychophysical experiments in tactile perception of these two types of virtual textures.

Force and vibration are two important factors that let a person distinguish the characteristics of objects and textures through touching. Lederman and Klatzky [6] demonstrated that spatially distributed forces are important for humans to perform a set of sensory tasks (determination of force threshold and spatial resolution) and perceptual tasks (roughness estimation and 2D bar orientation determination). On the other hand, vibration was also verified to be a critical factor to perform a series

of tactile feedback tasks by Kontarinis et al. [5] and Wellman et al. [13]. Okamura et al. [9] created a library to record the vibratory information of tapping on materials and stroking textures and then played back the simulated results on a force-feedback joystick. Weisenberger and Krier [14] compared human performance in tactile perception of surface textures using both vibratory and force feedback devices. The results from these researchers indicated that force and vibration are critical factors in human tactile perception.

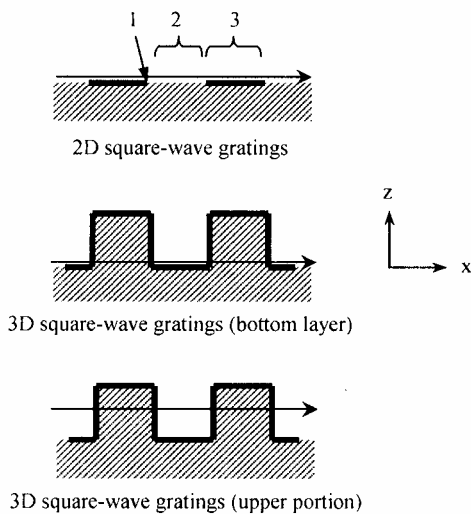
Many researchers have verified that virtual 2D texture alone can provide a compelling feeling of roughness and textures. Minsky [7] developed the 2D Sandpaper system to simulate haptic texture using only lateral force to construct textured surfaces that subjects scanned with a 2-degree-of-freedom (DOF) force-feedback joystick. She found that roughness perception was influenced heavily by contact force and that lateral force alone gave a compelling representation of haptic textures. Siira and Pai [12] discussed the decomposition of contact force in haptic texture and implemented only the lateral force to represent textured surfaces in their 2-DOF Pantograph haptic interface. Robles-De-La-Torre and Hayward [10] suggested the use of only planar forces to express 3D haptic shape. Their results indicated that explicit 3D geometry of a shape was not always necessary in the perception of a haptic shape.

Our study focuses on two types of textures – 2D and 3D square-wave gratings (see Figure 1). A 2D square wave is defined as rectangular gratings (50% duty cycle) parallel with one another *without* amplitude, whereas a 3D square wave is defined as rectangular gratings *with* amplitude. Our hypothesis is that 2D and 3D square-wave textures can provide an identical 3D tactile sensation. We will investigate under what circumstance this hypothesis is valid.

This study has potential benefit to both haptic interface design and haptic rendering techniques for virtual textures. If our hypothesis is valid, a 2-DOF haptic interface, which typically would possess a simpler

mechanical architecture, may be sufficient to convey realistic 3D tactile information for certain textures. Furthermore, because 2D square-wave gratings have fewer texture features than true 3D square-wave gratings, less computational effort would be required for rendering of virtual haptic textures.

The possible paths that a human hand can take across 2D and 3D square-wave textures are shown in Figure 1. For a 2D square wave, there is one general path. To feel the square-wave gratings, users need to move their hand on the flat textured surface. For a 3D square wave, there are two possible paths of contact movement. The first path is on the bottom layer of the textured surface. The second is across the upper portion of the texture, in and out of the gratings.



**Figure 1. Possible paths to move across 2D and 3D square-wave gratings (Regions 1, 2, and 3 are discussed in Sections 3.2 and 3.3)**

In this study, our approach was to measure vibration and force to see how they are related to the ability to distinguish between the two texture types. Human capability to distinguish between the two texture types was examined in texture discrimination experiments. Our study gives an initial step to understand how humans perceive 2D and 3D textures.

## 2. Theory

The mechanical properties of the human arm have been investigated by Hogan and Mussa-Ivaldi [3][8]. In their studies, a human arm, displaced by an apparatus from an equilibrium position, was moved back to its original position by the arm's restoring force. The

stiffness of the human arm was then derived from the relation between the restoring force and displacement.

In Hogan and Mussa-Ivaldi's numerical method, the arm stiffness for planar arm movement was represented by the matrix term in:

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} -K_{xx} & -K_{xy} \\ -K_{yx} & -K_{yy} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} \quad (1)$$

A linear force-displacement relation is assumed for small displacements ( $dx$  and  $dy$ ). The off-diagonal terms in the stiffness matrix ( $K_{xy}$  and  $K_{yx}$ ) can cause a displacement in one direction to exert a restoring force in another direction. These off-diagonal terms of the stiffness correspond to the interaction between the joints of the arm. Because of such off-diagonal terms in a more general three-dimensional stiffness matrix representing the arm, when the human hand is moved across periodic textures in x-direction, a restoring force may be generated in z-direction (normal to surface). Similar to the arm, the machine itself can also cause cross coupling to occur. As a result, the human operator may feel that the texture sticks out of the surface (3D tactile sensation) even though he/she does not move in the direction normal to the surface.

## 3. Vibration and Force Measurement

### 3.1. Measurement Setup

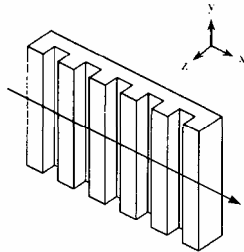


**Figure 2. Haptic interface used for experiments**

The haptic interface that we used permits three DOF translational motion and force at the human-machine interface (see Figure 2). The interface has a parallel configuration in which all heavy actuator housings are fixed to the common ground. This reduces both the weight and inertia of the moving mechanical system. Additionally, the parallel architecture is more rigid than machines with serial configurations. The nominal position resolution of the haptic interface is 0.016 mm.

Its continuous exertable force is 19 N; peak force is 69 N [2].

The haptic interface was controlled by a Pentium II 266 MHz PC running under MS-DOS. The sampling rate of the software program was 1 kHz for vibration measurement and 714 Hz for force measurement.



**Figure 3. Orientation of texture display and hand movement in vibration and force measurement**

During the experimental measurements, the 2D and 3D square-wave gratings were rendered vertically on the x-y plane when the hand was moved across the texture grating in positive x-direction from left to right at about 127 mm/s (5 inch/s) (see Figure 3). The stiffness of the texture was set to 2 N/mm. The periods of both 2D and 3D square-wave gratings were set to 5.08 mm (0.2 inch), whereas the peak-to-peak amplitude of 3D square-wave gratings was set to 50.8 mm (2 inch).

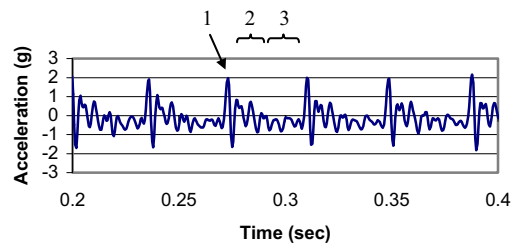
### 3.2. Vibration Measurement

Vibration is a cue that humans often use in tactile perception [5][13]. In vibration measurement, a 3-axial shear accelerometer (PCB Piezotronics, Inc.; Model No.: 356A16) was attached to the haptic interface at the link closest to the handgrip. A spectral analyzer (Agilent 35670A) was used to measure the haptic interface's acceleration in both time and frequency domains while the hand was moving across the texture gratings. During the measurement, the 2D and 3D square-wave gratings were displayed vertically. The subjects used their right hand to grasp the handle, maintaining their arm approximately in a horizontal plane, and moved the handle across the textures from left to right once (see Figure 8).

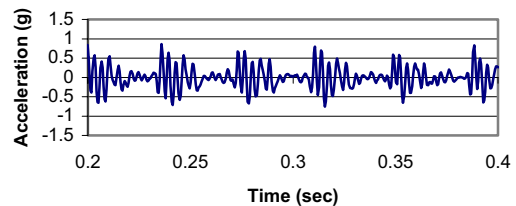
Sample acceleration measurements for 2D square-wave gratings are shown in Figure 4. In the time domain, the sharp rise of positive acceleration corresponded to the moment that the joystick interaction point has just left a bump because the sudden loss of resistant force made the sharp rise of acceleration in the positive x-direction (Region 1) (see Figures 1 and 4.1). After that, the linkage decelerated because the hand was moved at a constant velocity. As a result, the linkage acceleration fluctuated

as it reached steady state (Region 2). The linkage acceleration kept dropping as the hand was moved inside the next bump (Region 3) until another sharp rise of acceleration after the hand passed through the bump.

For the acceleration measurements in the z-direction (normal to the surface), big fluctuations of acceleration occurred after the sudden rise of acceleration in x-direction (see Figure 4.2). This big jump of acceleration depended on the virtual texture shape and stiffness. The fluctuations of acceleration in z-direction might provide the human operator a 3D tactile feeling because the vibration was in the direction normal to the surface.



1. X-Axis



2. Z-Axis

**Figure 4. Acceleration measurements in time domain when moving across 2D square-wave gratings**

Acceleration patterns for hand interaction with the bottom and upper portion of 3D square-wave gratings were very similar to those for 2D square-wave gratings. From the acceleration measurements, an observer would unlikely be able to distinguish between the two textures based on vibration cues when moving across the texture gratings.

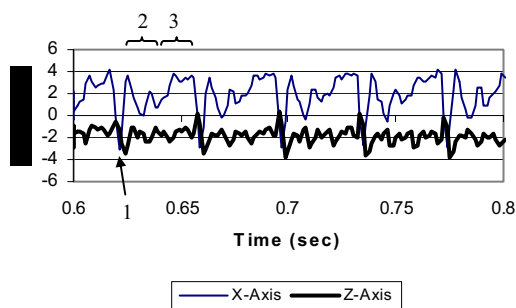
### 3.3. Force Measurement

Force is another main factor in tactile perception. The 6-axis force/torque sensor (ATI F/T Mini 10/20) built into the haptic interface below the handgrip was used to measure the force exerted at the hand. The 3-axis force measurements in transducer relative coordinates were

then converted to the fixed world coordinates of the haptic device.

Sample force measurements for hand interaction with 2D square-wave gratings are shown in Figure 5. The force profiles for 2D square-wave gratings can be explained using the study of Hajian and Howe [1] about the transient response of an outstretched human index finger. Hajian and Howe measured the force and acceleration for flexion-extension of the metacarpophalangeal (MCP) joint of an index finger and used the data to fit the parameters of their linear second-order, lumped-element model. The calculated values of the three force components, the inertial force ( $ma$ ), the damping force ( $bv$ ), and the stiffness force ( $kx$ ) were estimated individually and used to explain the behavior of their model of human finger. In their experiment, during the initial few milliseconds, inertial force was dominant. Next, the effects of stiffness and damping forces increased. Finally, at the end of the measurement, the stiffness force became the dominant term.

For 2D square-wave gratings, the x-axis component of the force profile is analyzed as follows (see Figures 5 and 1). After the joystick interaction point had just left the bump, the sudden loss of resistance in the direction of hand movement caused the x-coordinate force to drop sharply (Region 1). The first peak of force after the sudden drop was due to the inertia properties of the machine and hand. This observation follows from the results of Hajian and Howe. Later, after the first peak, both the acceleration,  $a$ , and the inertial force,  $ma$ , of the linkage oscillated left and right reaching steady state as the participant attempted to produce constant velocity hand movement (zero acceleration) (Region 2). As a result, the fluctuation of force measurements in x-axis was observed. The gradual rise of force in the positive x-direction observed as the hand moved further into the next bump was due to increasing stiffness force (Region 3).

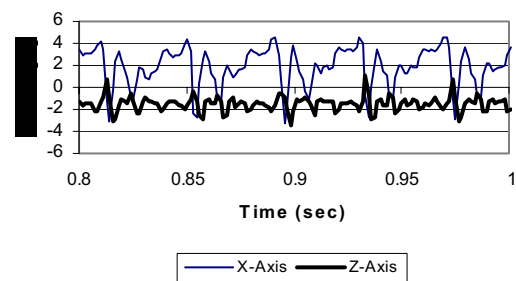


**Figure 5. Force measurements in time domain when moving across 2D square-wave gratings**

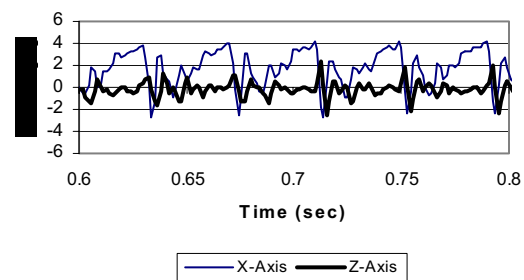
The force measurements in the z-axis (normal to the surface) showed a sharp change when a sudden drop of

force occurred in the x-axis, i.e., when the hand had just left a bump. Because the big change of force was in z-direction, the human operator would feel a force normal to the surface, contributing to a 3D tactile percept.

The force measurements of hand interaction with the bottom layer and upper portion of 3D square-wave gratings are shown in Figures 6 and 7. In general, there was no observable difference between x-axis force profiles when the hand was moved across 2D and 3D square-wave gratings. However, the z-axis force profiles of 2D square wave and 3D square wave (bottom layer) had an approximate 1.7-N mean offset from zero. This offset depended on how hard the hand pushed against the virtual surface in z-direction. On the other hand, the force profile of 3D square wave (upper portion) had no offset because the hand did not have contact with the bottom layer of the grating.



**Figure 6. Force measurements in time domain when moving across the bottom of 3D square-wave gratings**



**Figure 7. Force measurements in time domain when moving across the upper portion of 3D square-wave gratings**

In summary, it is unlikely that a person would feel the difference between 2D and 3D square-wave grating unless he/she interacts with the upper portion of the 3D square-wave. To interact with the upper portion of the grating, the person would need to pay close attention to the displacement offset in z-direction. However, rather than holding their hands in the air to interact only with the

upper portion of the 3D square-wave gratings, we observed that people tended to lean their hand against the surface and interact with the bottom layer of 3D square-wave gratings.

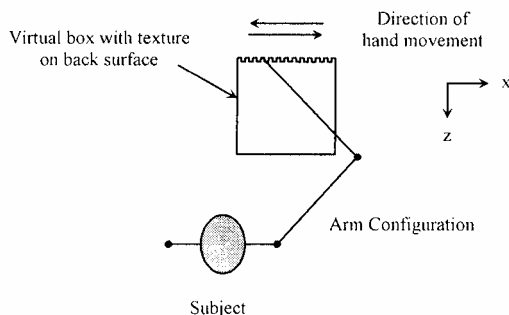
## 4. Tactile Perception Experiments

Along with the engineering measurement of vibration and force, we conducted psychophysical experiments to test whether human subjects can perceive any significant difference between interactions with 2D and 3D square-wave gratings. From the psychophysical results, we can discuss under what conditions the 2D and 3D square-wave gratings provide the same 3D tactile percept.

### 4.1. Procedure

Nine subjects (8 males and 1 female; age 18-39) with no reported tactile and visual impairment participated in all portions of this study. All used their right hand to interact with the texture gratings during the experiments.

Subjects used the same 3-DOF haptic interface and simulated haptic textures described above in the psychophysical study. The 2D and 3D square-wave gratings were displayed vertically on the inside back surface of a 127 x 127 x 127 mm (5 x 5 x 5 inch) virtual box (see Figure 8). In all the experiments, the subjects were allowed to move freely within the box. The texture stiffness was set to 2 N/mm, and the update frequency of the haptic interface controller was 1 kHz.



**Figure 8. Experimental setup of human tactile perception in comparison of 2D and 3D square-wave gratings (top view)**

This study consisted of three separate experiments to compare 2D and 3D square-wave gratings. The experiments differed from one another in the manner that the subjects interacted with the texture gratings. In each experiment, the period and peak-to-peak amplitude of the square-wave gratings could be either 0.51 mm (0.02 inch), 1.52 mm (0.06 inch), or 4.57 mm (0.18 inch). Each combination of period and amplitude was repeated

according to the method of constant stimuli in blocks of 20 for a total of 180 judgments. The order of the nine blocks was randomized in each experiment.

The subjects needed to respond whether pairs of sequentially displayed textures were the same. The first and the second textures could be either 2D or 3D square-wave gratings. Both textures had the same spatial period, while the amplitude of the 3D texture was varied. For each amplitude-period combination, half (10) of the presentations were “catch” trials, in which the stimuli pairs were the same.

Before the start of the actual experiments, all subjects were allowed to explore the 2D and 3D square-wave gratings with the period and amplitude between 0.51 mm (0.02 inch) and 4.57 mm (0.18 inch) in their own ways until they noticed the difference between them. Then the subjects were required to complete a practice run before starting the actual study.

The required interaction with the texture gratings was different in the three experiments. In Experiment 1, the subject was paced by a position indicator on the computer screen to move his/her hand across the texture gratings. In each test, when the first texture appeared, the subject needed to move his/her hand from the left end to the right end of the box in 1 s and then return to the left end in the next 1 s. Then the second texture appeared, and the subject needed to make the same left-right-left paced movement. After that, the subject judged whether or not the two textures were identical.

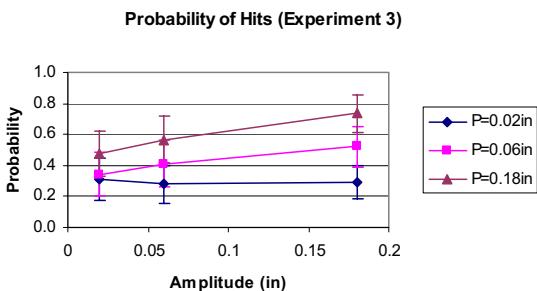
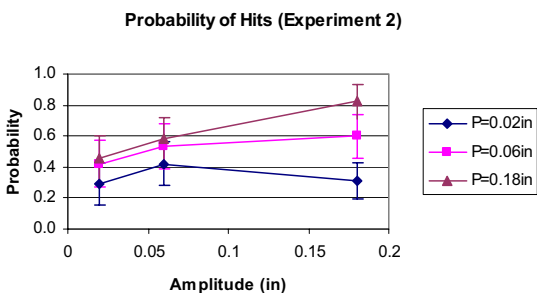
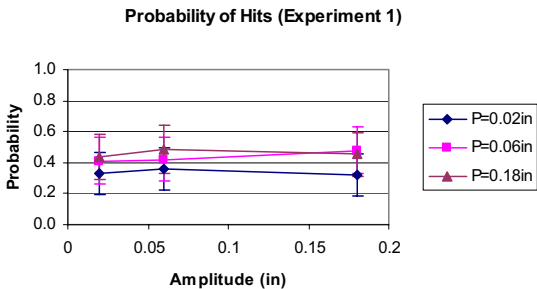
In Experiment 2, the subjects were free to explore the virtual gratings using any motion pattern they chose. They were no longer required to follow the pace of the position indicator. In this experiment, each texture was displayed for 4 s for each test. The subject was required to make a judgment after the two textures were displayed.

In Experiment 3, the subject was required to follow the pace of the position indicator again. However, to move across the texture gratings this time, the subject was instructed to use a predetermined strategy described to him/her by the experiment monitor. This strategy was for the subject to move his/her hand toward himself/herself by a small amount in the positive z-direction until the subject felt the resistance of the first bump of the square-wave gratings (see Figure 8). Because the subject needed to move his/her hand a short distance toward himself/herself in order to clear the first bump, the subject might be expected to notice the peak-to-peak amplitude of the square-wave gratings.

### 4.2. Results and Discussion

Detection theory was used to do the data analysis of the experimental results [4]. The theory utilizes the relationship between two types of response, hit and false

alarm. The term “hit” refers to a correct judgment when a stimulus or stimulus difference is present; the term “false alarm” refers to the incorrect judgment that a stimulus or stimulus difference is present on a “catch trial.”

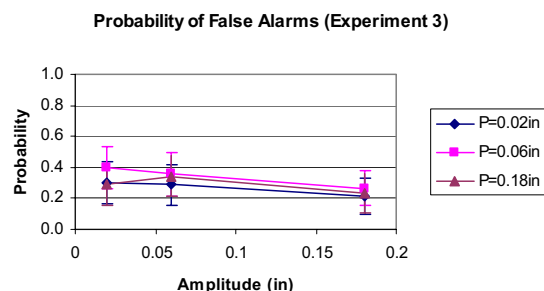
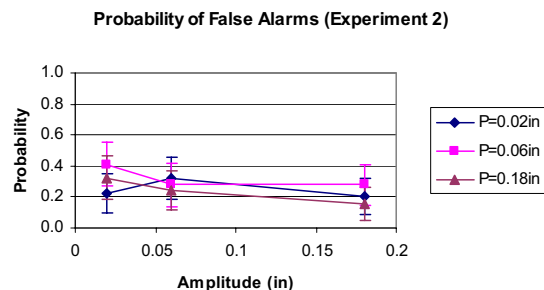
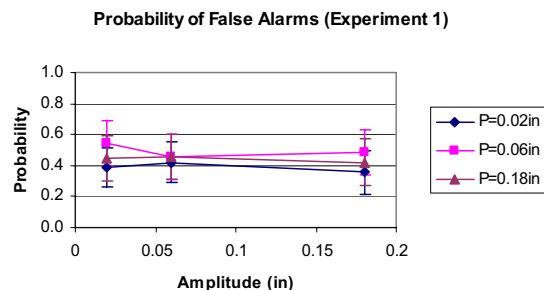


**Figure 9. Hit rates for all three experiments of tactile perception in discrimination between 2D and 3D square-wave gratings**

The experimental results of the hit and false alarm rates are shown in Figures 9 and 10. The error bars of the data points were determined from the standard errors of binomial distribution [11]. From Experiment 1, in which hand motion was paced by the position indicator, it appears that subjects were unable to detect the difference between the 2D and 3D square-wave gratings because the proportions of hits remained at ~0.4 (below the 0.5 level expected for equiprobable random guessing), regardless of the spatial period and amplitude of the square-wave gratings.

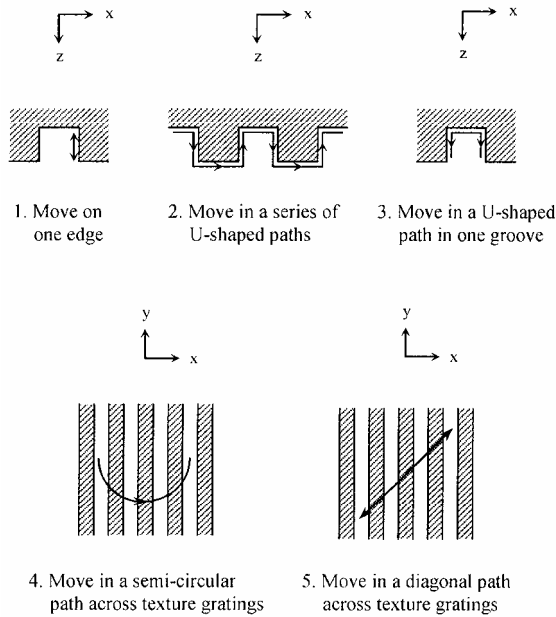
In Experiment 2, most subjects could notice the difference between the 2D and 3D square-wave gratings

when both spatial period and amplitude of the texture gratings were big. For example, when the square-wave gratings were at 0.18-inch period and 0.18-inch peak-to-peak amplitude, the average proportion of hits was as high as  $0.82 \pm 0.11$ , and the average proportion of false alarms was as low as  $0.16 \pm 0.11$ . From the graph of hit rate, the proportion of hits tended to increase with amplitude for a given period. It also tended to increase with period for a given amplitude. When the amplitude of the square-wave gratings was small, most subjects could not feel the difference between the 2D and 3D square-wave gratings, resulting in the low proportion of hits. Similarly, when the period of the square-wave gratings was low, subjects could pass through the texture gratings easily with very little resistance, resulting in the low proportion of hits.



**Figure 10. False alarm rates for all three experiments of tactile perception in discrimination between 2D and 3D square-wave gratings**





**Figure 11. Different touching strategies of the subjects used to distinguish between 2D and 3D square-wave gratings in Experiment 2**

We asked the subjects to describe how they developed their own methods for exploring the 2D and 3D square-wave gratings in Experiment 2. Three subjects moved up on the edge of the square-wave gratings in order to feel the depth (or amplitude) of the texture (see Figure 11.1). Three subjects moved in a series of U-shaped paths for two to three bumps (see Figure 11.2). One subject moved in a U-shaped path back and forth along the three edges in a single groove of the texture (see Figure 11.3). Finally, one subject moved in a semi-circular path across the texture gratings (see Figure 11.4), and another subject moved back and forth in diagonal direction across the texture (see Figure 11.5). From their experimental results, the seven subjects who touched the texture by the methods in Figures 11.1–11.3 did better than the subjects who touched the texture using the methods in Figures 11.4–11.5. In particular, these seven subjects showed a higher proportion of hits and a lower proportion of false alarms when they interacted with textures that had big period and amplitude. On the other hand, the remaining two subjects did not seem to use an effective method to find the difference between 2D and 3D square-wave gratings. For example, in the combination of 0.18-inch period and 0.18-inch amplitude in Experiment 2, the mean proportion of hits was 0.82 and the mean proportion of false alarms was 0.16. However, in this combination, two poorer performing subjects had hit rates of 0.7 and 0.5 and false alarm rates of 0.3 and 0.2, respectively. Therefore, the strategy used

to explore the texture gratings affected the outcome of the subjects' results in Experiment 2.

In Experiment 3, where the subjects were instructed to move their hand toward themselves slightly in order to feel the edge of the first bump, 6 out of 9 subjects could tell the difference between 2D and 3D square-wave gratings for the cases that both the spatial period and amplitude of the texture gratings were big. Furthermore, Figure 9 (Experiment 3) indicates that the proportion of hits increased with grating amplitude and period.

In summary, from the results of Experiments 2 and 3, the accuracy of distinguishing between 2D and 3D square-wave gratings increased with increasing amplitude and increasing period. Also, the exploration strategy used to interact with the square-wave gratings affected subject's ability to determine whether the texture was 2-dimensional or 3-dimensional. From the results of Experiment 1, subjects could not tell the difference between 2D and 3D square-wave textures when moving their hand across the gratings at about 127 mm/s (5 inch/s). However, when the subjects had an opportunity to move up over the edges of the square-wave gratings as in Experiments 2 and 3, most of them (6 out of 9 subjects in both experiments) could notice the difference between 2D and 3D square-wave gratings for the textures with the largest peak-to-peak amplitude (0.18 inch) and period (0.18 inch). Nevertheless, they still could not notice the difference between the two textures when either the spatial period or amplitude was below 0.06 inch.

## 5. Conclusions

Based on the engineering measurements of vibration and force and the results of haptic perception experiments, a person is unlikely to feel the difference between 2D and 3D square-wave gratings when moving his/her hand across the texture gratings if either the spatial period or amplitude is below 0.06 inch for the texture stiffness of 2 N/mm. From the measurements, the vibration and force profiles for the interaction with 2D and 3D square-wave gratings were similar. The only difference between the two texture gratings was the elimination of offset force normal to the surface which occurred when hand interaction was restricted to the upper portion of 3D square-wave gratings. There was no observable difference in force profile when the hand interacted with the 2D square-wave gratings and the bottom portion of the 3D square-wave gratings. Nevertheless, from tactile perception experiments, if the spatial amplitude and period of the square-wave gratings are big enough, the person may notice the difference between the two textures when moving his/her hand up slowly on the edges of the square-wave gratings. Furthermore, our tactile perception experiments indicated

that the accuracy of distinguishing between 2D and 3D square-wave gratings increased with increasing amplitude and increasing period.

From these results, if a person's hand is moved across the texture gratings in a manner similar to our experimental setup, or if the period or peak-to-peak amplitude of the square-wave texture is small (<0.06 inch), a 2D square-wave texture is sufficient to model a true 3D square-wave texture. A person may not be able to notice the difference between the two textures unless his/her hand is moved slowly up on the edges of the texture gratings for a big texture. However, from our observation, people seldom touched the texture gratings by moving up on the edges of texture; instead, they tended to keep leaning against the texture surface while moving across the gratings. Our results support the observations from Minsky and Robles-De-La-Torre that the lateral forces alone can effectively simulate surface textures and can be used to emulate the experience of touching true 3D textures [7][10], provided the grating size remains below a specified certain threshold.

## 6. Acknowledgment

This research was supported at UC Berkeley by NASA Ames Research Center Cooperative Agreement NCC2-1255 with funding from the NASA Intelligent Systems Human Centered Computing Program and the NASA Space Human Factors Engineering Program.

## Reference

- [1] Hajian, A. and Howe, R., "Identification of the mechanical impedance of human fingers," *Proceedings of the ASME Dynamic Systems and Control Division*, DSC-Vol. 55-1, ASME, 1994, pp. 319–327.
- [2] Ho, P., "Haptic Simulation of 3D Primitive Objects and Textures Using a Novel Force-Reflecting Interface," Ph.D. Dissertation, Department of Mechanical Engineering, U.C. Berkeley, 2002.
- [3] Hogan, N., "The mechanics of multi-joint posture and movement control," *Biological Cybernetics*, Vol. 52, 1985, pp. 315–331.
- [4] Kling, J. W. and Riggs, L. A., *Woodworth & Schlosberg's Experimental Psychology*, Holt, Rinehart and Winston, Inc., 3<sup>rd</sup> ed., pp. 11–46.
- [5] Kontarinis, D. A. and Howe, R. D., "Tactile display of vibratory information in teleoperation and virtual environments," *Presence*, Vol. 4, No. 4, 1995, pp. 387–402.
- [6] Lederman, S. J. and Klatzky, R. L., "Designing haptic interfaces for teleoperational and virtual environments: should spatially distributed forces be displayed to the fingertip?" *Proceedings of the ASME Dynamic Systems and Control Division*, DSC-Vol. 61, ASME, 1997, pp. 11–15.
- [7] Minsky, M. and Lederman, S. J., "Simulated haptic textures: roughness," *Proceeding of the ASME Dynamic Systems and Control Division*, DSC-Vol. 58, 1996, pp. 421–426.
- [8] Mussa-Ivaldi, F. A., Hogan, N., and Bizzi, E., "Neural, mechanical, and geometric factors subserving arm posture in humans," *Journal of Neuroscience*, Vol. 5, 1985, pp. 2732–2743.
- [9] Okamura, A. M., Dennerlein, J. T., and Howe, R. D., "Vibration feedback models for virtual environments," *Proceedings of the IEEE International Conference on Robotics and Automation*, Leuven, Belgium, 1998, pp. 674–679.
- [10] Robles-De-La-Torre, G. and Hayward, V., "Virtual surfaces and haptic shape perception," *Proceedings of the ASME Dynamic Systems and Control Division*, DSC-Vol. 69-2, ASME, 2000, pp. 1081–1085.
- [11] Sachs, L., *Applied Statistics: A Handbook of Techniques*, Springer-Verlag New York, 1984, pp. 162–164.
- [12] Siira, J. and Pai, D. K., "Haptic texturing: a stochastic approach," *Proceedings of the IEEE International Conference on Robotics and Automation*, Minneapolis, MN, 1996, pp. 557–562.
- [13] Wellman, P. and Howe, R. D., "Towards realistic display in virtual environments," *Proceedings of the ASME Dynamic Systems and Control Division*, DSC-Vol. 57-2, ASME, 1995, pp. 713–718.
- [14] Weisenberger, J. M. and Krier, M. J., "Haptic perception of simulated surface textures via vibratory and force feedback displays," *Proceedings of the ASME Dynamic Systems and Control Division*, DSC-Vol. 61, ASME, 1997, pp. 55–60.